

LÍMITES ALGEBRAICOS RESUELTOS

Ejercicio 1

$$\lim_{x \rightarrow 1} \frac{x^2 + 2x - 3}{x^2 - 5x + 4}$$

Respuesta:

$$\lim_{x \rightarrow 1} \frac{(x + 3)(x - 1)}{(x - 4)(x - 1)}$$

$$\lim_{x \rightarrow 1} \frac{1 + 3}{1 - 4}$$

$$-4/3$$

Ejercicio 2

$$\lim_{x \rightarrow 1} \frac{x^4 - x^5}{1 - x}$$

Respuesta:

$$\lim_{x \rightarrow 1} \frac{x^4(1 - x)}{1 - x}$$

$$\lim_{x \rightarrow 1} x^4$$

$$1$$

Ejercicio 3

$$\lim_{x \rightarrow 2} \frac{2 - \sqrt[2]{x + 2}}{x - 2}$$

Respuesta:

$$\lim_{x \rightarrow 2} \frac{2 - \sqrt[2]{x + 2}}{x - 2} \cdot \frac{2 + \sqrt[2]{x + 2}}{2 + \sqrt[2]{x + 2}}$$

$$\lim_{x \rightarrow 2} \frac{-(x - 2)}{(x - 2)(2 + \sqrt[2]{x + 2})}$$

$$\lim_{x \rightarrow 2} \frac{-1}{2 + \sqrt[2]{x + 2}}$$

$$\frac{-1}{2 + \sqrt[2]{4}}$$

- 1 / 4

Ejercicio 4

$$\lim_{x \rightarrow -2} \frac{5x^4 - 3x^2 - 68}{2x^5 - 3x^2 + 2x + 8}$$

Respuesta:

$$\lim_{x \rightarrow -2} \frac{(x + 2)(5x^3 - 10x^2 + 17x - 34)}{(x + 2)(2x^4 + 8x^2 - 19x + 40)}$$
$$\frac{-74}{87}$$

Ejercicio 5

$$\lim_{x \rightarrow -2} \frac{x^3 + 3x^2 + 2x}{x^2 - x - 6}$$

Respuesta:

$$\lim_{x \rightarrow -2} \frac{x(x + 1)(x + 2)}{(x + 2)(x - 3)}$$
$$\lim_{x \rightarrow -2} \frac{x(x + 1)}{x - 3}$$
$$\frac{-2(-2 + 1)}{-2 - 3}$$
$$\frac{2}{-5}$$

Ejercicio 6

$$\lim_{x \rightarrow 9} \frac{x^2 - 81}{\sqrt{x} - 3} = \lim_{x \rightarrow 9} \frac{(x - 9)(x + 9)}{\sqrt{x} - 3} \cdot \frac{\sqrt{x} + 3}{\sqrt{x} + 3}$$
$$= \lim_{x \rightarrow 9} \frac{(x - 9)(x + 9)(\sqrt{x} + 3)}{x - 9} = \lim_{x \rightarrow 9} (x + 9)(\sqrt{x} + 3)$$
$$= (9 + 9)(\sqrt{9} + 3) = (18)(6) = 108$$

Ejercicio 7

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{x}{\sqrt{1+3x}-1} &= \lim_{x \rightarrow 0} \frac{x}{\sqrt{1+3x}-1} \cdot \frac{\sqrt{1+3x}+1}{\sqrt{1+3x}+1} \\ &= \lim_{x \rightarrow 0} \frac{(x)(\sqrt{1+3x}+1)}{(1+3x)-1} = \lim_{x \rightarrow 0} \frac{(x)(\sqrt{1+3x}+1)}{3x} = \lim_{x \rightarrow 0} \frac{\sqrt{1+3x}+1}{3} \\ &= \frac{\sqrt{1+3(0)}+1}{3} = \frac{\sqrt{1}+1}{3} = \frac{2}{3} \end{aligned}$$

Ejercicio 8

$$\lim_{x \rightarrow 0} \frac{\sqrt{(2-t)} - \sqrt{2}}{t}$$

$$\lim_{x \rightarrow 0} \frac{\sqrt{(2-t)} - \sqrt{2}}{t} = \frac{\sqrt{(2-0)} - \sqrt{2}}{0} = \frac{\sqrt{2} - \sqrt{2}}{0} = \frac{0}{0}$$

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{\sqrt{(2-t)} - \sqrt{2}}{t} &= \lim_{x \rightarrow 0} \frac{(\sqrt{(2-t)} - \sqrt{2})(\sqrt{(2-t)} + \sqrt{2})}{t(\sqrt{(2-t)} + \sqrt{2})} \\ &= \lim_{x \rightarrow 0} \frac{(\sqrt{(2-t)})^2 - (\sqrt{2})^2}{t(\sqrt{(2-t)} + \sqrt{2})} = \lim_{x \rightarrow 0} \frac{(2-t) - 2}{t(\sqrt{(2-t)} + \sqrt{2})} \\ &= \lim_{x \rightarrow 0} \frac{-t}{t(\sqrt{(2-t)} + \sqrt{2})} = \lim_{x \rightarrow 0} \frac{-1}{\sqrt{(2-t)} + \sqrt{2}} = \frac{-1}{\sqrt{(2-0)} + \sqrt{2}} \\ &= \frac{-1}{\sqrt{2} + \sqrt{2}} = \frac{-1}{2\sqrt{2}} \end{aligned}$$

Ejercicio 9

$$\lim_{x \rightarrow 0} \frac{x}{\sqrt{(1+3x)} - 1}$$

RESPUESTA:
evaluando:

$$\lim_{x \rightarrow 0} \frac{x}{\sqrt{(1+3x)} - 1} = \frac{0}{\sqrt{(1+3(0))} - 1} = \frac{0}{\sqrt{1} - 1} = \frac{0}{0}$$

$$\begin{aligned}
 \lim_{x \rightarrow 0} \frac{x}{\sqrt{1+3x} - 1} &= \lim_{x \rightarrow 0} \frac{x(\sqrt{1+3x} + 1)}{(\sqrt{1+3x} - 1)(\sqrt{1+3x} + 1)} \\
 &= \lim_{x \rightarrow 0} \frac{x(\sqrt{1+3x} + 1)}{(\sqrt{1+3x})^2 - 1} = \lim_{x \rightarrow 0} \frac{x(\sqrt{1+3x} + 1)}{(1+3x) - 1} \\
 &= \lim_{x \rightarrow 0} \frac{x(\sqrt{1+3x} + 1)}{3x} = \lim_{x \rightarrow 0} \frac{(\sqrt{1+3x} + 1)}{3} = \frac{(\sqrt{1+3(0)} + 1)}{3} \\
 &= \frac{(\sqrt{1} + 1)}{3} = \frac{2}{3}
 \end{aligned}$$

Ejercicio 10

$$\begin{aligned}
 &\lim_{x \rightarrow -2} \frac{x^3 + 2x^2 + 1 - 1}{x + 2} \\
 &\lim_{x \rightarrow -2} \frac{x^3 + 2x^2}{x + 2} \\
 &\lim_{x \rightarrow -2} \frac{x^2(x + 2)}{x + 2} \\
 &\lim_{x \rightarrow -2} x^2 \\
 &4
 \end{aligned}$$

Ejercicio 11

$$\lim_{x \rightarrow 1} \frac{x - 1}{\sqrt{x^2 + 3} - 2}$$

Respuesta:

$$\begin{aligned}
 &\lim_{x \rightarrow 1} \frac{x - 1}{\sqrt{x^2 + 3} - 2} \cdot \frac{\sqrt{x^2 + 3} + 2}{\sqrt{x^2 + 3} + 2} \\
 &\lim_{x \rightarrow 1} \frac{(x - 1)(\sqrt{x^2 + 3} + 2)}{(x - 1)(x + 1)} \\
 &\frac{\sqrt{4} + 2}{2} \\
 &\frac{4}{2} \\
 &2
 \end{aligned}$$

Ejercicio 12

$$\lim_{x \rightarrow a} \frac{x^2 - a^2}{x^2 - 2ax + a^2}$$

Respuesta:

$$\lim_{x \rightarrow a} \frac{(x + a)(x - a)}{(x - a)(x - a)}$$

$$\lim_{x \rightarrow a} \frac{x + a}{x - a}$$

Ejercicio 13

$$\lim_{x \rightarrow 4} \frac{x - 4}{x^2 - x - 12}$$

Respuesta:

$$\lim_{x \rightarrow 4} \frac{x - 4}{(x - 4)(x + 3)}$$

$$\lim_{x \rightarrow 4} \frac{x - 4}{x + 3}$$

1 / 7

Ejercicio 14

$$\lim_{x \rightarrow 3} \frac{x^3 - 27}{x^2 - 9}$$

Respuesta:

$$\lim_{x \rightarrow 3} \frac{(x - 3)(x^2 + 3x + 9)}{(x - 3)(x + 3)}$$

$$\lim_{x \rightarrow 3} \frac{x^2 + 3x + 9}{x + 3}$$

9 / 2

Ejercicio 15

$$\lim_{h \rightarrow 0} \frac{(x + h)^2 - x^2}{h}$$

Respuesta:

$$\lim_{h \rightarrow 0} \frac{x^2 + 2hx + h^2 - x^2}{h}$$

$$\lim_{h \rightarrow 0} 2x + h$$

2x

Ejercicio 16

$$\lim_{x \rightarrow \infty} \frac{3x - 2}{8x + 7}$$

Respuesta:

$$\lim_{x \rightarrow \infty} \frac{3 - (2/x)}{8 + (7/x)} \quad 3/8$$

Ejercicio 17

$$\lim_{x \rightarrow \infty} \frac{6x^2 + 2x + 1}{5x^2 - 3x - 4}$$

Respuesta:

$$\lim_{x \rightarrow \infty} \frac{6 + (2/x) + (1/x^2)}{5 - (3/x) - (4/x^2)} \quad 6/5$$

Ejercicio 18

$$\lim_{x \rightarrow \infty} \frac{x^2 + x - 2}{4x^3 - 1}$$

Respuesta:

$$\lim_{x \rightarrow \infty} \frac{(1/x) + (1/x^2) - (2/x^3)}{4 - (1/x^3)} \quad 0$$

Ejercicio 19

$$\lim_{x \rightarrow \infty} \frac{2x^3}{x^2 + 1}$$

Respuesta:

$$\lim_{x \rightarrow \infty} \frac{2}{(1/x) + (1/x^3)} \quad n.e$$

Ejercicio 20

$$\begin{aligned} \lim_{x \rightarrow 4} f(x) &= \lim_{x \rightarrow 4} \frac{\sqrt[2]{x} - 2}{x - 4} \\ \lim_{x \rightarrow 4} f(x) &= \lim_{x \rightarrow 4} \frac{(\sqrt[2]{x} - 2)(\sqrt[2]{x} + 2)}{(x - 4)(\sqrt[2]{x} + 2)} \\ \lim_{x \rightarrow 4} f(x) &= \lim_{x \rightarrow 4} \frac{x - 4}{(x - 4)(\sqrt[2]{x} + 2)} \cdot \lim_{x \rightarrow 4} f(x) = \lim_{x \rightarrow 4} \frac{1}{\sqrt[2]{x} + 2} \\ \lim_{x \rightarrow 4} f(x) &= \frac{1}{4} \end{aligned}$$

Ejercicio 21

47) $\lim_{x \rightarrow 5} \frac{x^2 - 2x - 35}{x^2 + 3x - 10}$

Respuesta:

$$\begin{aligned} \lim_{x \rightarrow 5} \frac{(x - 7)(x + 5)}{(x + 5)(x - 2)} \\ \frac{-5 - 7}{-5 - 2} \\ \frac{-12}{-7} \end{aligned}$$

Ejercicio 22

$$\lim_{x \rightarrow 5} \frac{1 - \sqrt[2]{x - 4}}{x - 5} \cdot \frac{1 + \sqrt[2]{x - 4}}{1 + \sqrt[2]{x - 4}}$$

Respuesta:

$$\begin{aligned} \lim_{x \rightarrow 5} \frac{-(x - 5)}{(x - 5)(1 + \sqrt[2]{x - 4})} \\ \frac{-1}{1 + \sqrt[2]{x - 4}} \\ \frac{-1}{1 + \sqrt{1}} \\ \frac{-1}{2} \end{aligned}$$

Ejercicio 23

$$49) \lim_{x \rightarrow 6} \frac{\sqrt[2]{x-2} - 2}{x-6} \frac{\sqrt[2]{x-2} + 2}{\sqrt[2]{x-2} + 2}$$

Respuesta:

$$\lim_{x \rightarrow 6} \frac{x-6}{(x-6)(\sqrt[2]{x-2} + 2)}$$

$$\lim_{x \rightarrow 6} \frac{1}{(\sqrt[2]{x-2} + 2)}$$

$$\frac{1}{\sqrt{4} + 2}$$

$$\frac{1}{4}$$

Ejercicio 24

$$50) \lim_{h \rightarrow 0} \frac{(x+h)^3 - x^3}{h}$$

Respuesta:

$$\lim_{h \rightarrow 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h}$$

$$\lim_{h \rightarrow 0} \frac{h(3x^2 + 3xh + h^2)}{h}$$

$$\lim_{h \rightarrow 0} 3x^2 + \lim_{x \rightarrow 0} 3xh + \lim_{h \rightarrow 0} h^2$$

$$3x^2$$

Ejercicio 25

$$51) \lim_{x \rightarrow 2} \frac{4-x^2}{3-\sqrt{x^2+5}} \frac{3+\sqrt{x^2+5}}{3+\sqrt{x^2+5}}$$

Respuesta:

$$\lim_{x \rightarrow 2} \frac{(4-x^2)(3+\sqrt{x^2+5})}{4-x^2}$$

$$\lim_{x \rightarrow 2} 3 + \sqrt{x^2+5}$$

$$3+3$$

Ejercicio 26

$$52) \lim_{h \rightarrow 0} \frac{\sqrt[2]{x+h} - \sqrt[2]{x}}{h}$$

Respuesta:

$$\begin{aligned} & \lim_{h \rightarrow 0} \frac{\sqrt[2]{x+h} + \sqrt[2]{x}}{\sqrt[2]{x+h} + \sqrt[2]{x}} \cdot \frac{h}{h} \\ & \lim_{h \rightarrow 0} \frac{h}{h(\sqrt[2]{x+h} + \sqrt[2]{x})} \\ & \lim_{h \rightarrow 0} \frac{1}{\sqrt[2]{x+h} + \sqrt[2]{x}} \\ & \frac{1}{2\sqrt[2]{x}} \end{aligned}$$

$$= 3 + 3 = 6$$